Physics II
ISI B.Math
Midterm Exam : September 10, 2014

Total Marks: 80
Time : 3 hours

Answer all questions

1.(**Marks** $= 2 \times 8 = 16$)

For each of the following statements indicate whether it is true or false and give a brief explanation. Just a few lines should suffice.

(i) A Carnot engine whose thermal efficiency is very high is particularly well suited for a refrigerator, when run in the reverse direction.

(ii) A Carnot engine is operated using a gas of photons with an equation of state $P = (1/3)u$ where P is the pressure and u is the internal energy density. The change of Helmholz free energy $F = U - TS$ over a cycle of this engine is equal to zero.

(iii) An ideal monatomic gas is enclosed in a box with adiabatic walls. One of the walls has a valve, which when opened allows the gas to freely expand into an adjacent evacuated box which also has adiabatic walls. $< v^2 >$ of the molecules of the gas decreases on opening the valve.

(iv) $T\left(\frac{\partial S}{\partial T}\right)_P - T\left(\frac{\partial S}{\partial T}\right)_V$ is always equal to R.

(v) The entropy of an ideal gas is a function of temperature only.

(vi) For any system undergoing a thermodynamic process, the second law of thermodynamics requires that the entropy change of the system be greater than or equal to zero.

(vii) In a phase transition process of conversion of saturated vapour into liquid, the entropy and the Gibbs potential remain constant.

(viii) A body at temperature T_1 is brought in contact with a body at temperature T_2 where $T_1 > T_2$. The entropy of the universe increases in this process.

2. (**Marks** $= 7 + 2 + 7 = 16$)

A gas which obeys the van der Waals equation $(p + \frac{a}{V^2}) \times (V - b) = RT$ has a molar specific heat at constant volume C_V which is constant and independent of temperature. Consider a thermally insulated rigid container which is divided into two compartments (of volumes V_1 and V_2 respectively) separated by a valve which is initially closed. One mole of the above gas is introduced into the chamber of volume V_1 and its temperature T_1 noted. The valve is then opened and the gas expands to fill both the compartments.
(a) Find the final temperature T_2 of the gas after the valve is opened.
(b)What would be the final temperature if the real gas were replaced by an ideal gas?

(c) If the valve were replaced by a piston and the gas was quasistatically expanded to volume V_1+V_2 what would be the final temperature ?

3. (**Marks** $= 4+6+6 = 16$)

(a) State the Second Law of Thermodynamics in the Kelvin-Planck and Clausius form respectively.
(b) Show that if the Kelvin-Planck Statement is violated, it implies a violation of the Clausius statement.
(c) By considering a Carnot engine operating in the region of equilibrium coexistence of a liquid and its saturated vapour, derive the Clausius-Clapeyron equation given below. [Hint: On the P-V diagram for this system, draw two neighbouring isothermals at temperatures T and $T+dT$ respectively and operate the engine between these two isothermals by connecting them with adiabatic curves]

$$\left(\frac{dP}{dT}\right) = \frac{\lambda}{T(v_2 - v_1)}$$

where λ is the latent heat of the liquid - vapour phase transition, v_2 , v_1 are the specific volumes of the vapour and water respectively at the transition temperature.

4. (**Marks** $= 5+3+4+4 = 16$)

(a) From the fact that dV/V is an exact differential, derive the relation

$$\left(\frac{\partial\beta}{\partial P}\right)_T = -\left(\frac{\partial\kappa}{\partial T}\right)_P$$

where β is the coefficient of volume expansion and κ is the isothermal compressibility.

(b) Show that the third law of thermodynamics requires that the specific heat of a substance at constant volume must go to zero as $T \to 0$. Does an ideal gas obey the third law of thermodynamics ?

(c) A measure of the result of an adiabatic free expansion is provided by the Joule coeficient $\eta = \left(\frac{\partial T}{\partial V}\right)_U$, Show that $\eta = -\frac{1}{C_V}\left(\frac{\beta T}{\kappa} - P\right)$

(d) A measure of the Joule-Thomson expansion (adiabatic throttling process of isenthalpic expansion) is provided by the Joule-Thomson coefficient $\mu = \left(\frac{\partial T}{\partial P}\right)_H$. Show that $\mu = \frac{V}{C_P}(\beta T - 1)$.

5. (**Marks** $= 3+4+5+4 = 16$)

Two identical objects A and B are mechanically and thermally isolated from the rest of the world. Their initial temperatures are $\tau_A > \tau_B$. Each object has heat capacity C(the same for both objects) which is independent of temperature.

(a) Suppose the objects are placed in contact with each other and allowed to come to equilibrium. What is their final temperature ? How much work on the outside world is done in the process?

(b) Now suppose objects A (temperature τ_A) and B (temperature $\tau_B < \tau_A$) are used as the high and low temperature reservoirs of a heat engine. The engine extracts energy from object A (lowering its temperature), does work on the outside world and dumps waste heat to object B, (raising its temperature). When A and B are at the same temperature T_f the process is finished. What is the total work done by the engine ?

(c) What is the total change in entropy of the universe due to the process in (b) ? Show that if the process is consistent with the second law of thermodynamics, $T_f \geq \sqrt{\tau_A \tau_B}$.

(d) Show that the maximum amount of work obtainable from this engine is $C(\sqrt{\tau_A} - \sqrt{\tau_B})^2$.

Information you may or may not need:

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$$

$$\left(\frac{\partial x}{\partial y}\right)_f \left(\frac{\partial y}{\partial z}\right)_f \left(\frac{\partial z}{\partial x}\right)_f = 1$$

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$$

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$$

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$$